Moody's
ANALYTICS

Modelling interest rates in 2014

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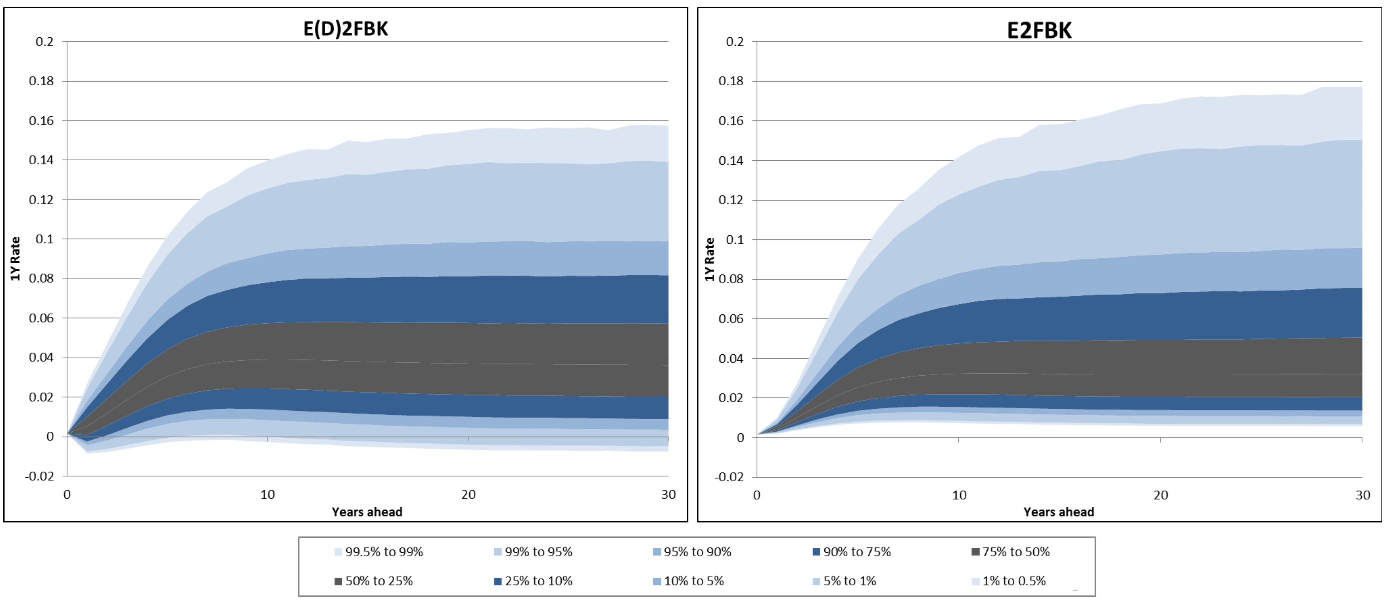
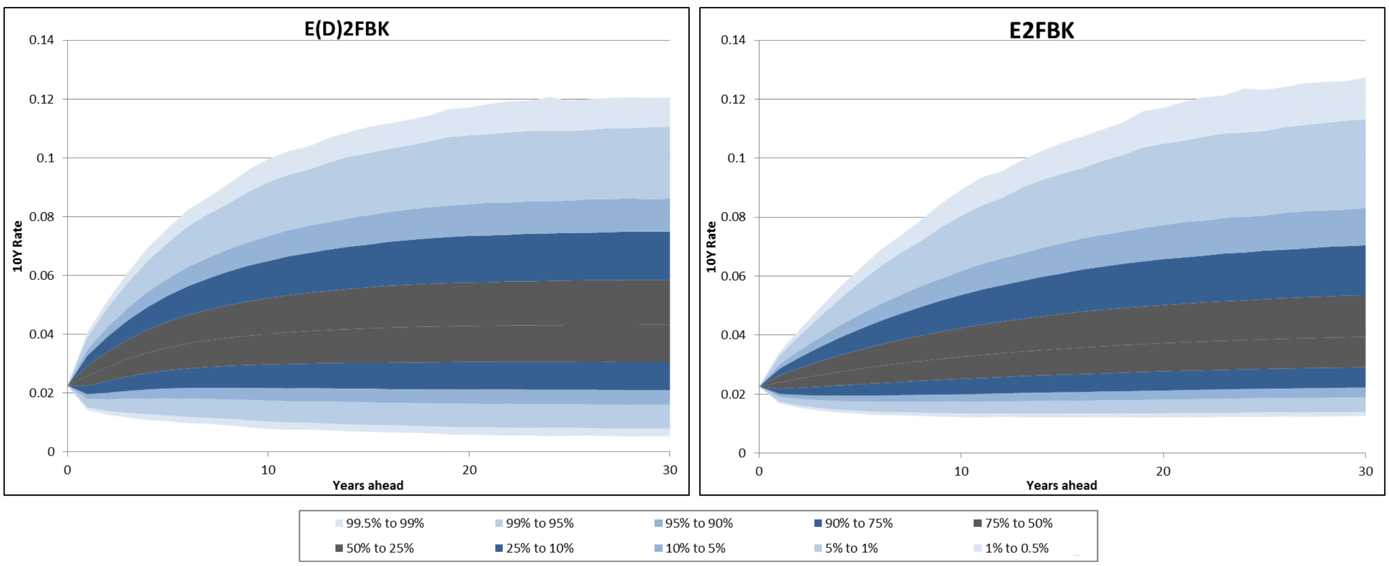
Nov, 2014

Model comparison E2FBK and E(D)2FBK, 1Y rates



Model comparison E2FBK and E(D)2FBK, 10Y rates



Overview

1. Key features and stylised facts for interest rate modelling
2. Modelling proportional volatility for interest rates
3. Model/calibration research and development

Interest rate modelling: stylised facts

Stylized facts for yield curve modelling

1. **Mean reversion in the long run:** Another way of saying that interest rates are stationary.

2. **Proportional volatility:** When interest rates are high the potential for a large subsequent change is bigger (same applies for low rates).

3. **Multi factor:** Different maturities along the yield curve exhibit imperfect correlation implying that multiple factors drive yield curve movements.

4. **Arbitrage free**: yield curves should be projected using arbitrage free models.

5. **Stochastic volatility:** Like other asset classes interest rates are subject to temporary bouts of extreme instability.

6. **Negative nominal rates are rare but possible:** In several of the world's major markets negative nominal rates have been observed.

Interest rates are mean-reverting in the long term

This is a fundamental feature underpinning numerous models of interest rates.

- In the *long term* we assume that interest rates will mean revert. **This may not be the case over shorter horizons.**
- This is the same as the assumption that interest rates are stationary in the long term.

Consider the process:

$$y_t = \alpha + \rho y_{t-1} + e_t$$

Non stationary if $\rho = 1$

- H_0: $\rho = 1$ (nonstationary/not mean reverting)
- H_1: $\rho < 1$ (stationary/mean reverting)

We can test this by estimating:

$$y_t - y_{t-1} = \alpha + \beta y_{t-1} + e_t$$

And testing the hypotheses:

$$H_0: \beta = 0$$
$$H_1: \beta < 0$$

This is the well known *Dickey Fuller test* for stationarity.

The parameters can also be interpreted as a measure of mean reversion, consider the discretized Vasicek model for interest rates

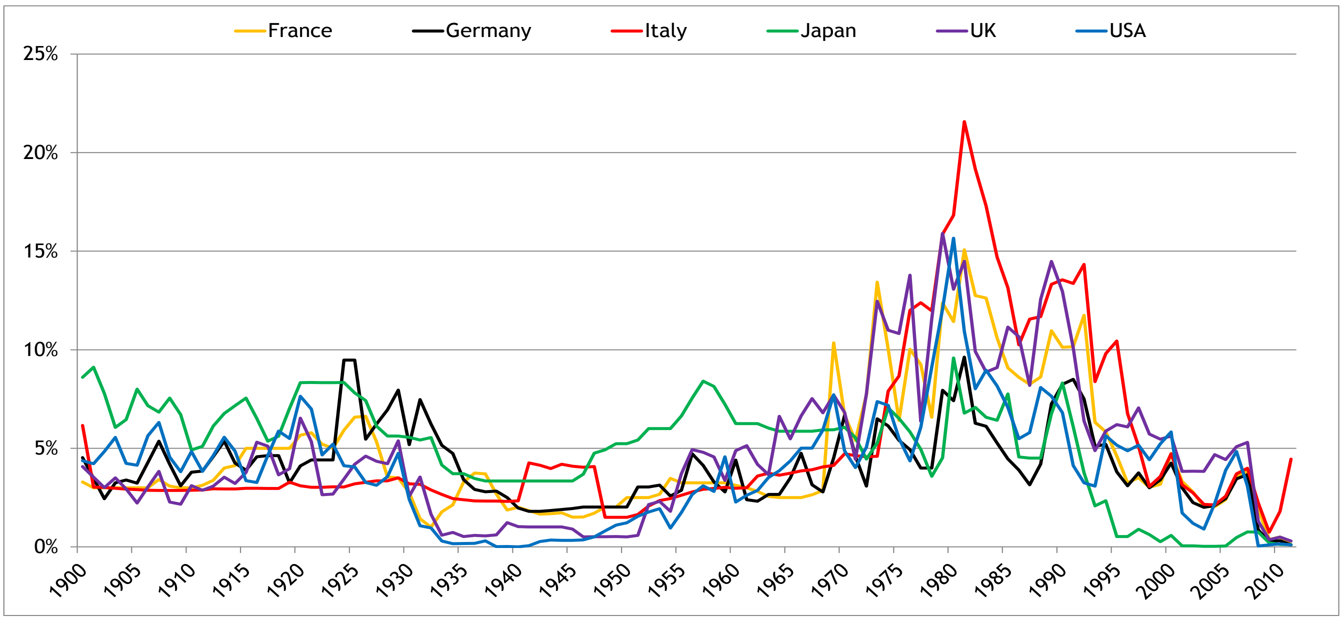
$$r_t - r_{t-1} = \theta(\mu - r_{t-1}) + \varepsilon_t$$

This can be re written

$$r_t - r_{t-1} = \theta\mu - \theta r_{t-1} + \varepsilon_t$$

We can test for this using long time series data

DF critical values are -2.58 at 10%, -2.89 at 5%, and -3.51 at 1%



	France	Germany	Italy	Japan	UK	USA
Beta	-0.12	-0.21	-0.05	-0.07	-0.11	-0.11
Constant	0.01	0.01	0.00	0.00	0.00	0.00
Std Err Beta	0.05	0.06	0.03	0.04	0.04	0.05
Dickey-Fuller statistic	-2.52	-3.44	-1.62	-1.93	-2.41	-2.41

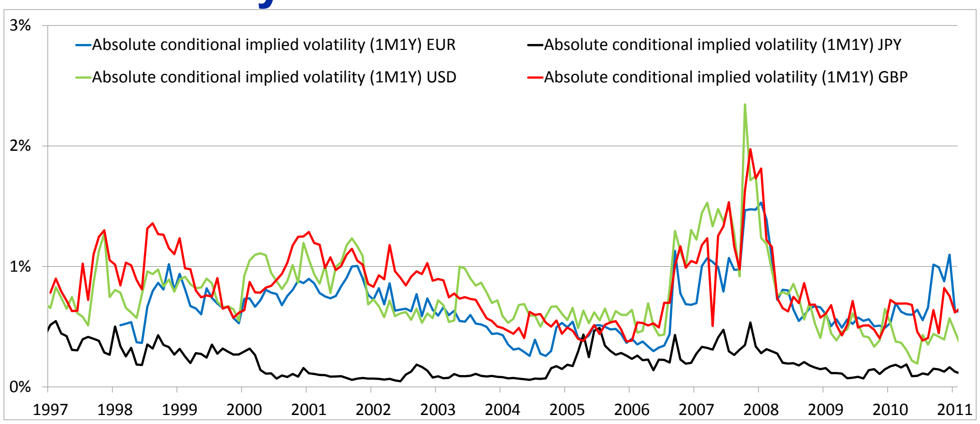
» From a purely statistical/econometric perspective interest rates are not *obviously* stationary! (Although we have used a fairly basic model that does not account for proportional volatility)

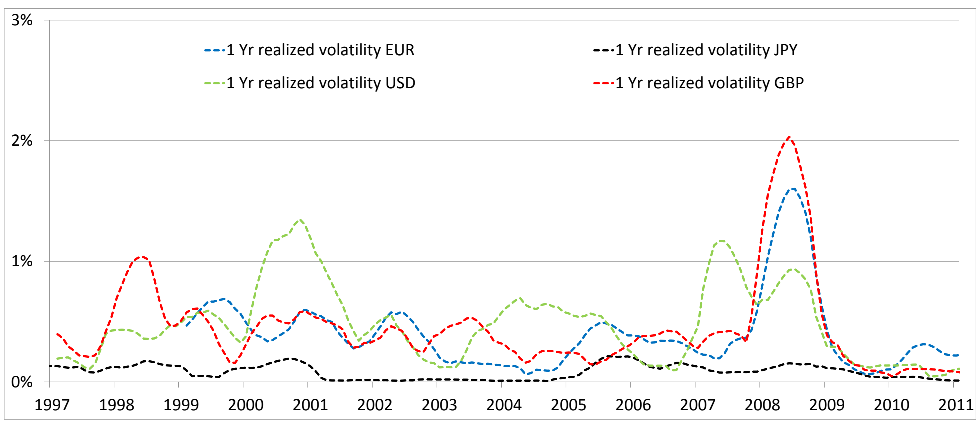
» We would assert that for fundamental reasons interest rates are stationary; the statistical evidence is not strong enough to imply that this is not the case (only just reject hypothesis of stationarity for some economies with simple model)

» Important implications for one year stresses; hard to justify rapid mean reversion especially if this implies a very large term premium

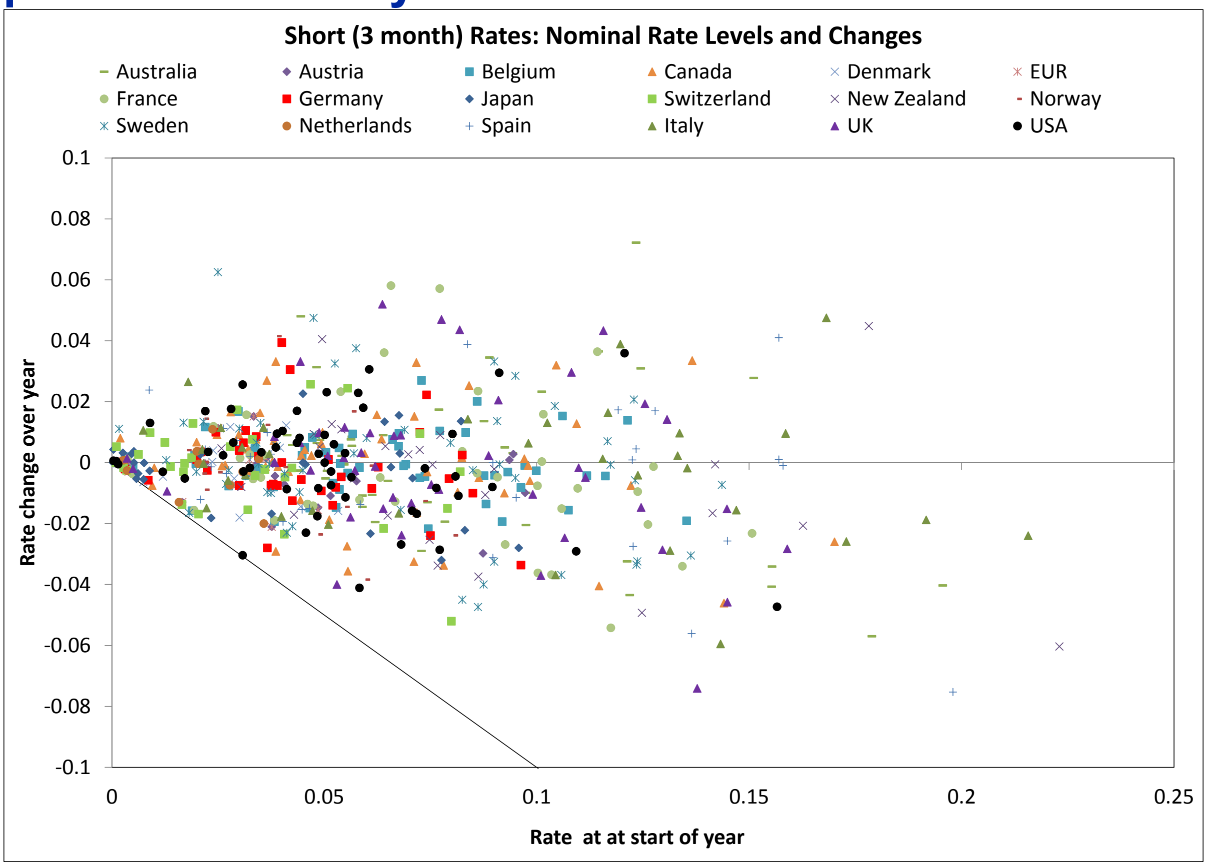
  » However; given the current economic environment and central bank policies is it sensible to assume some sort of near term unwinding and correction?

Stochastic volatility





Proportional volatility



Negative nominal interest rates

Traditional axiom of nominal interest rate modelling: *interest rates are bounded below at zero*

Closely related to approaches that model shocks as proportional to rate levels

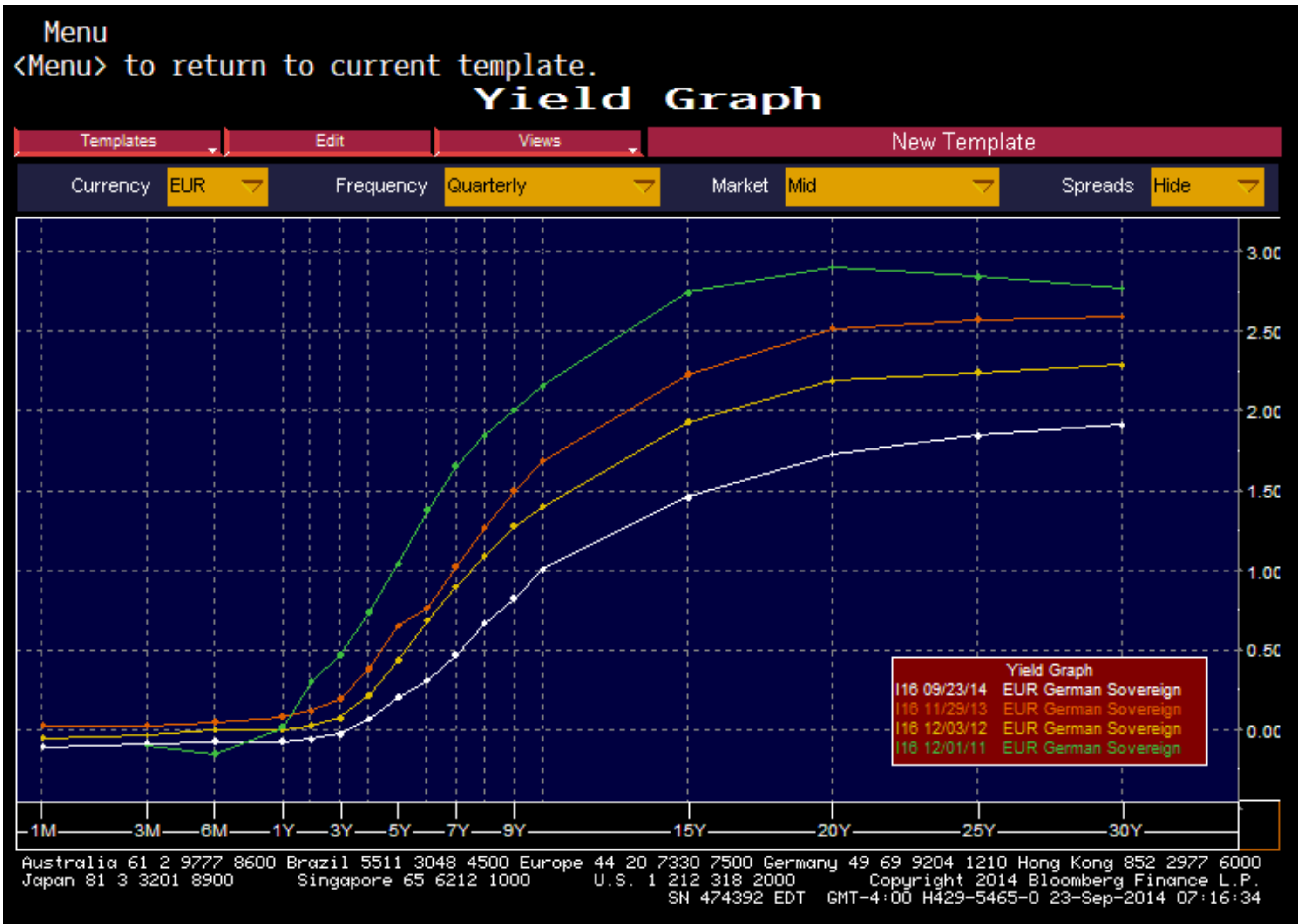
Fed rates have gone negative at some point in every year since 2008

- End 2008: 3 month T Bills trade at negative yields
- March 26th 2009: 1 month T Bills -0.015%
- January 27th 2010: 1 month T Bills -0.0101%
- August 4th 2011 : 1 month T Bills -0.0102%

Swiss and German yields have also gone negative

Some possible views:

1. Very short rates can move to be effectively zero/be very slightly negative for a short time; we might want to model a point probability of rates hitting zero/some slightly negative rate.
2. Negative rates are a potential future CB policy, therefore we would want to model a significant probability of negative yield curves.



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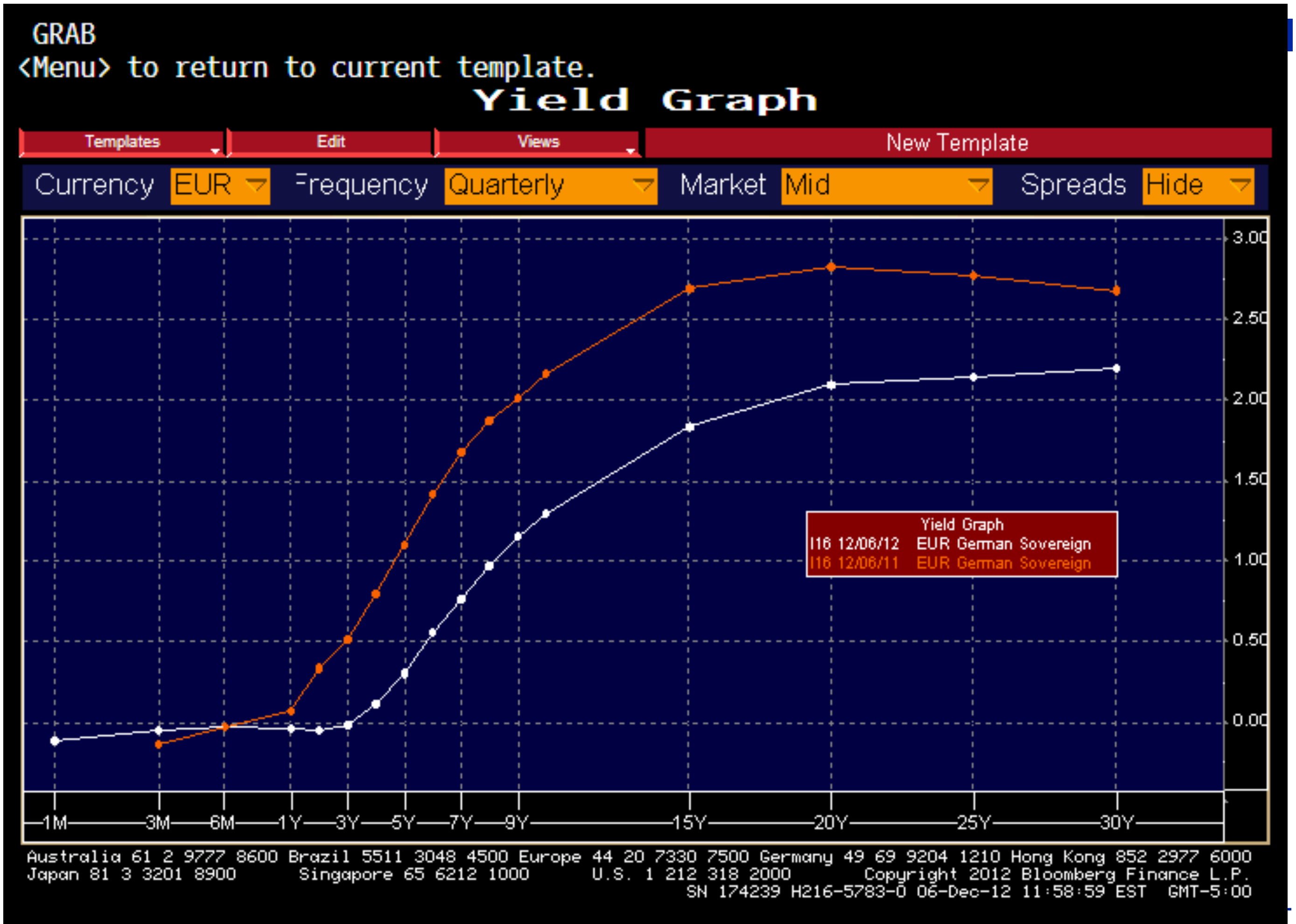

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Interest rate distributions

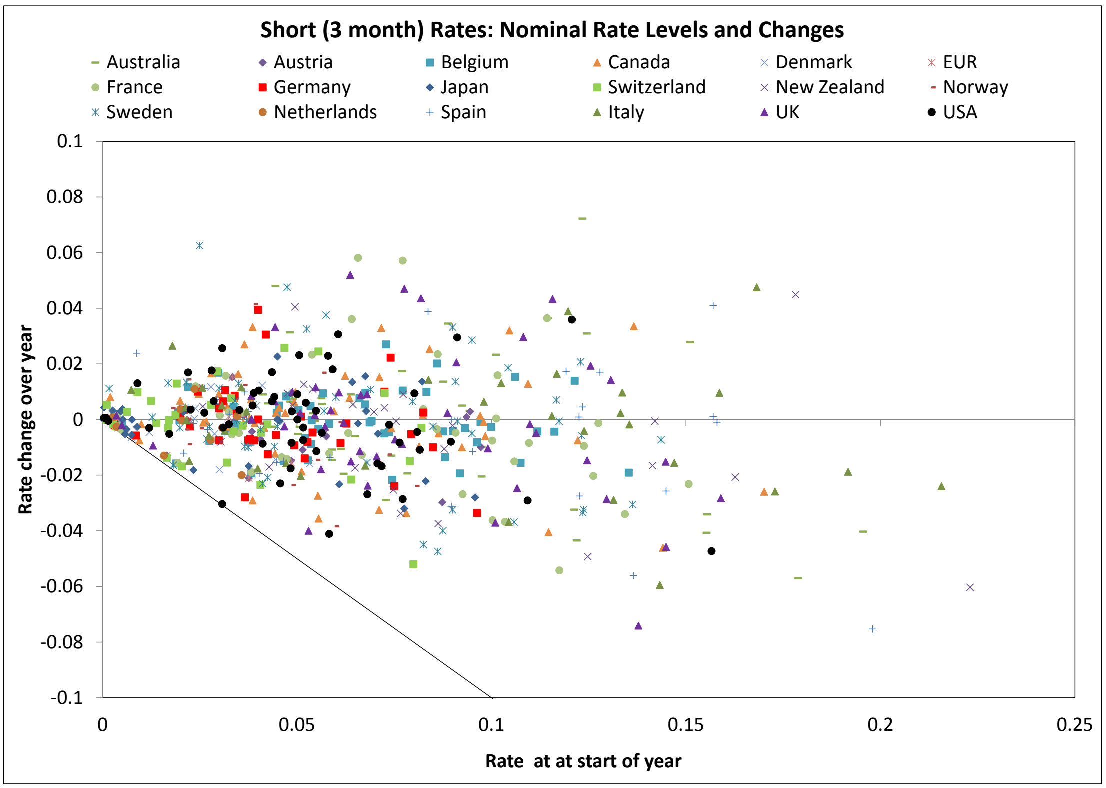
Modelling volatility; key considerations

Area of significant academic focus (Chan et al. (1992), Bliss and Smith (1998), Rebonato et al. (2012)).

- Do I want to have the option of modelling negative rates?
- Should volatility tend to zero as rates tend to zero?
  - Standard Formula: "*Irrespective of the stress factors, the absolute change of interest rates in the downward scenario should at least be one percentage point. Where the unstressed rate is lower than 1% the shocked rate in the downward scenario should be assumed to be 0%.*" (EIOPA Technical Specifications Part 1, SCR.5.24)
- Is volatility stochastic?
- How should *proportional volatility* be modelled? Lognormal, "Constant elasticity of variance"…?

Proportional volatility

» Key feature of any nominal interest rate model



MOODY'S ANALYTICS

Interest rate models; a number of approaches to proportional volatility (1)

- Various approaches to enhancing lognormal stochastic models to capture the relationship between the state variable and its volatility.
  - Generally developed in the context of pricing ‘skew’ in option prices (the observed ‘smile’ when implied volatility is plotted against the strike of the option)
- ‘constant elasticity of variance’ models have often been favoured for nominal interest rate modelling
  - No negative rates (traditionally seen as an advantage)
  - Zero volatility when rates are zero
  - Degree of elasticity can control higher moments of the distribution
  - Higher volatility when rates are low than lognormal model

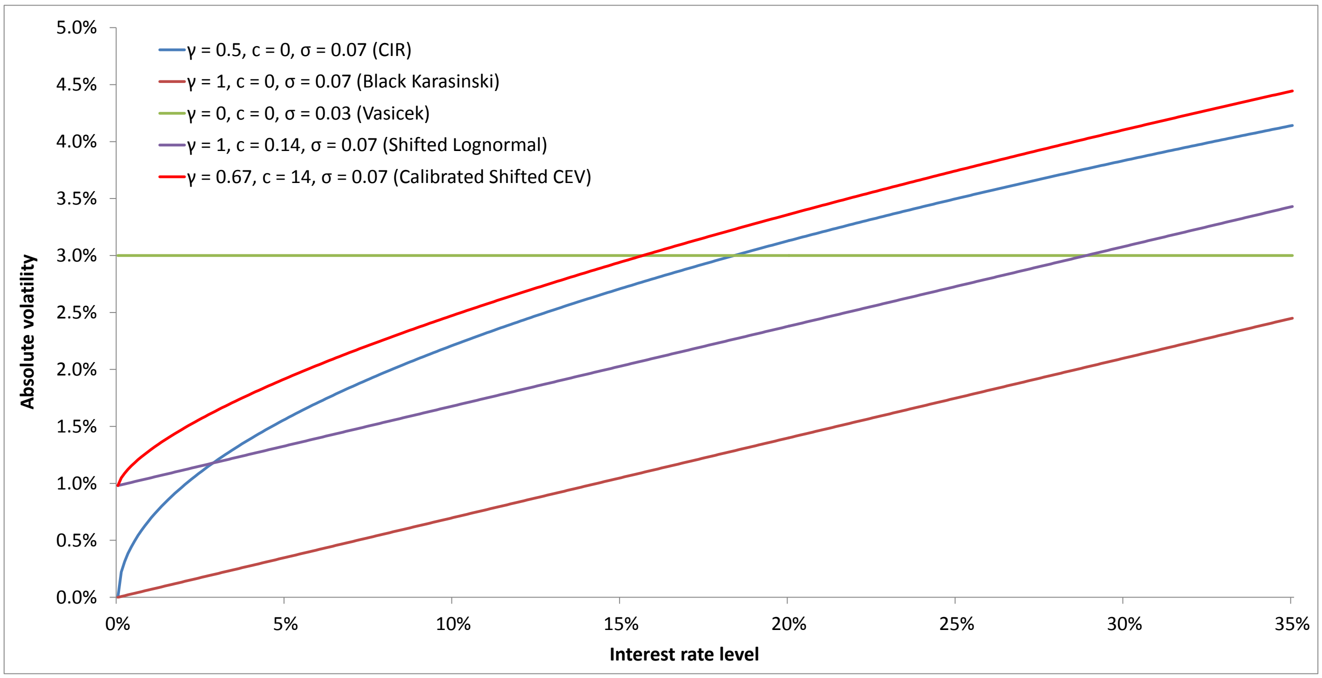
$$dr = (a + br)dt + \sigma(r^{\gamma})dZ$$

- Displacement of the lognormal modelled state variable (a shift) can also produce similar distribution behaviour
  - Produces negative rates
  - Non zero volatility when rates are at zero

$$dr = (a + br)dt + \sigma(r + c)dZ$$

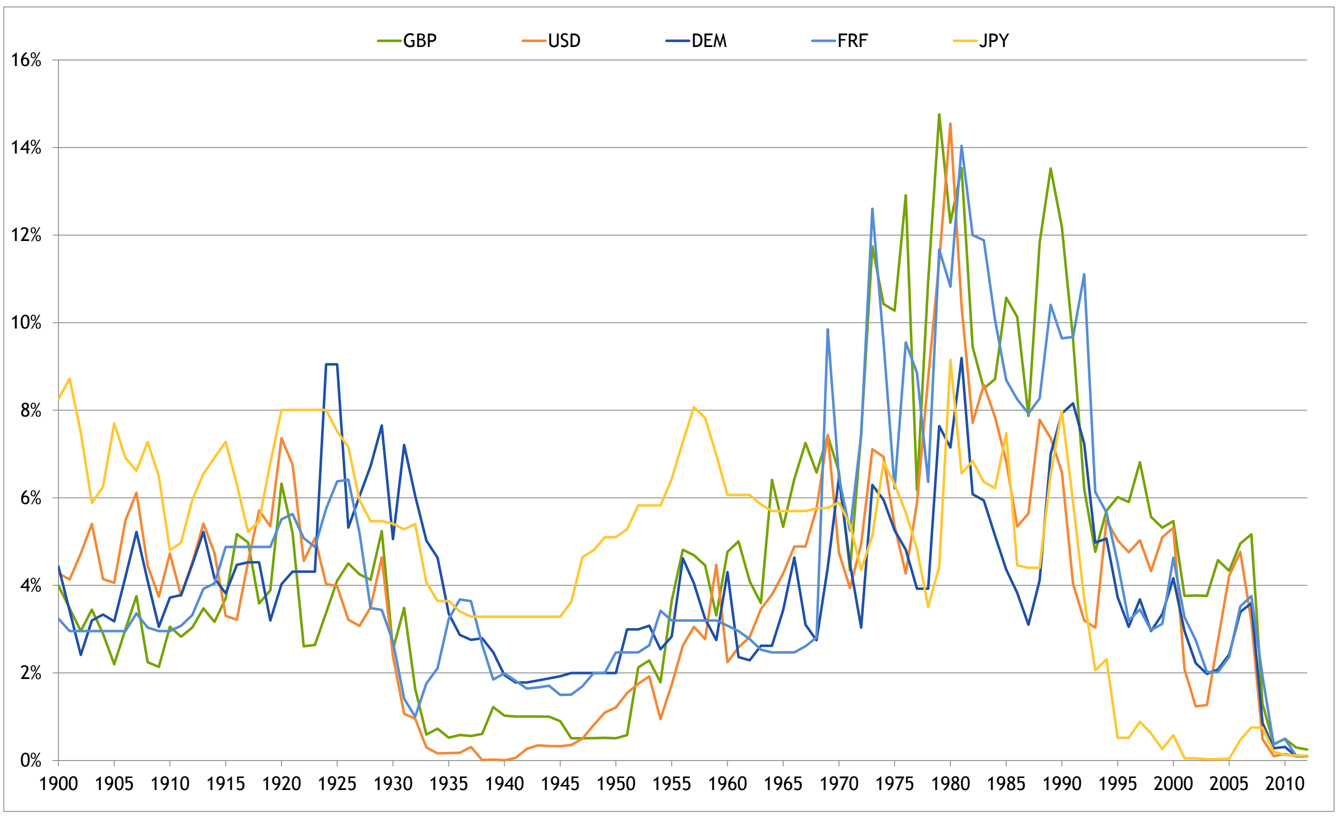
Interest rate models; a number of approaches to proportional volatility (2)

$$dr = (a + br)dt + \sigma(r^{\gamma} + c)dZ$$



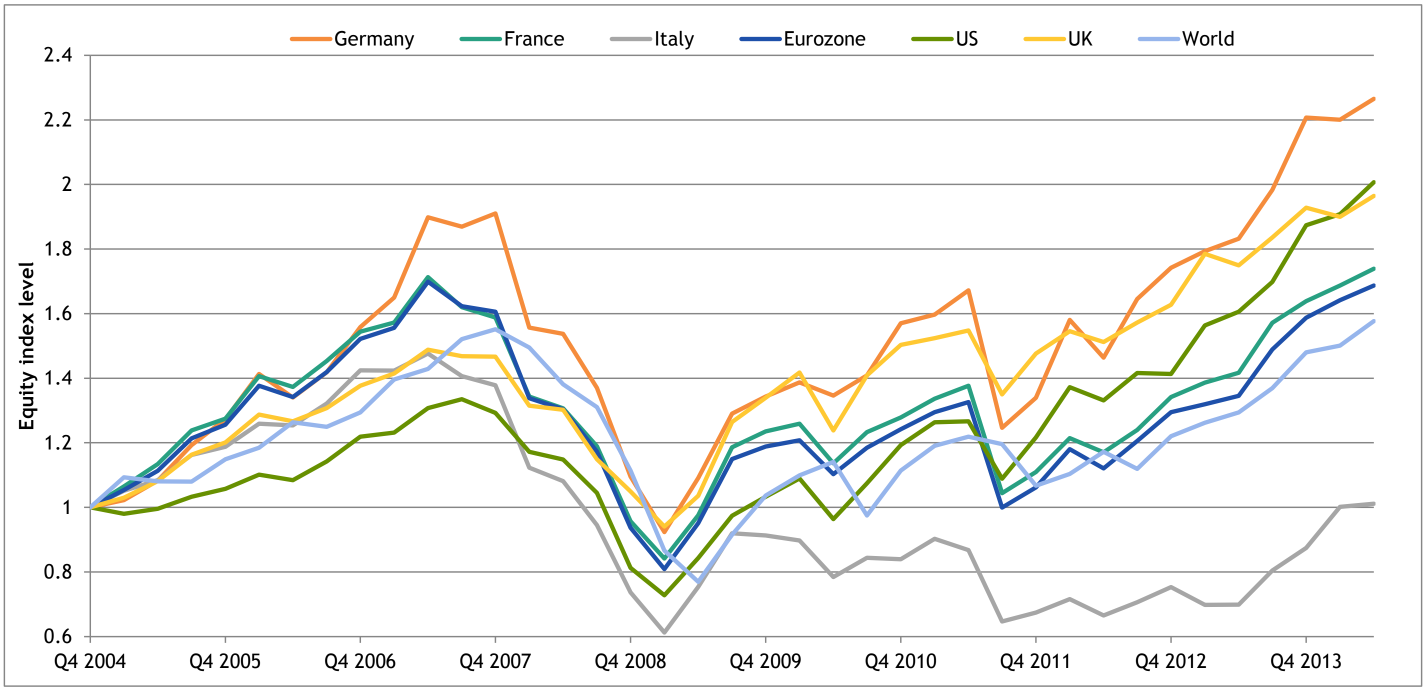
Moody's Analytics nominal rate modelling research

Long term view (20th century short rates)



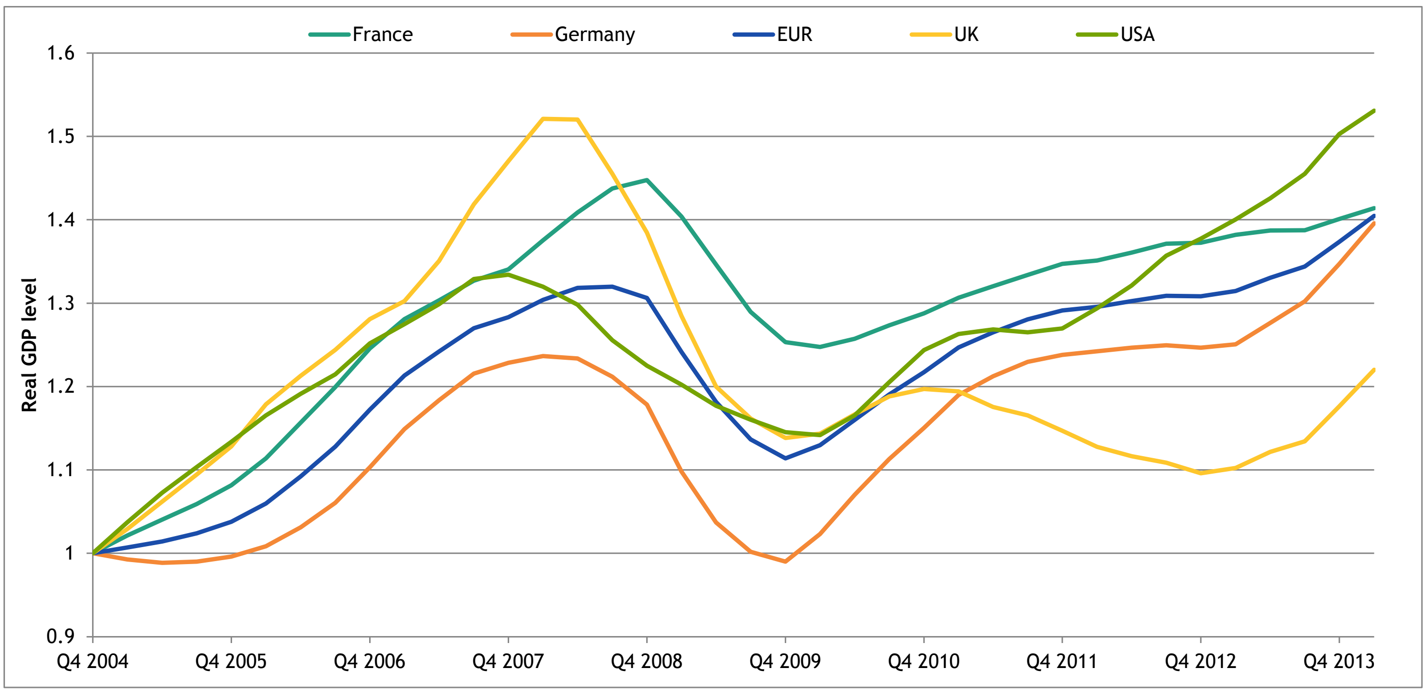
2007/2008 financial crisis and subsequent recession (1)

» Significant liquidity shocks to global financial markets
  – Herding and “animal spirits”



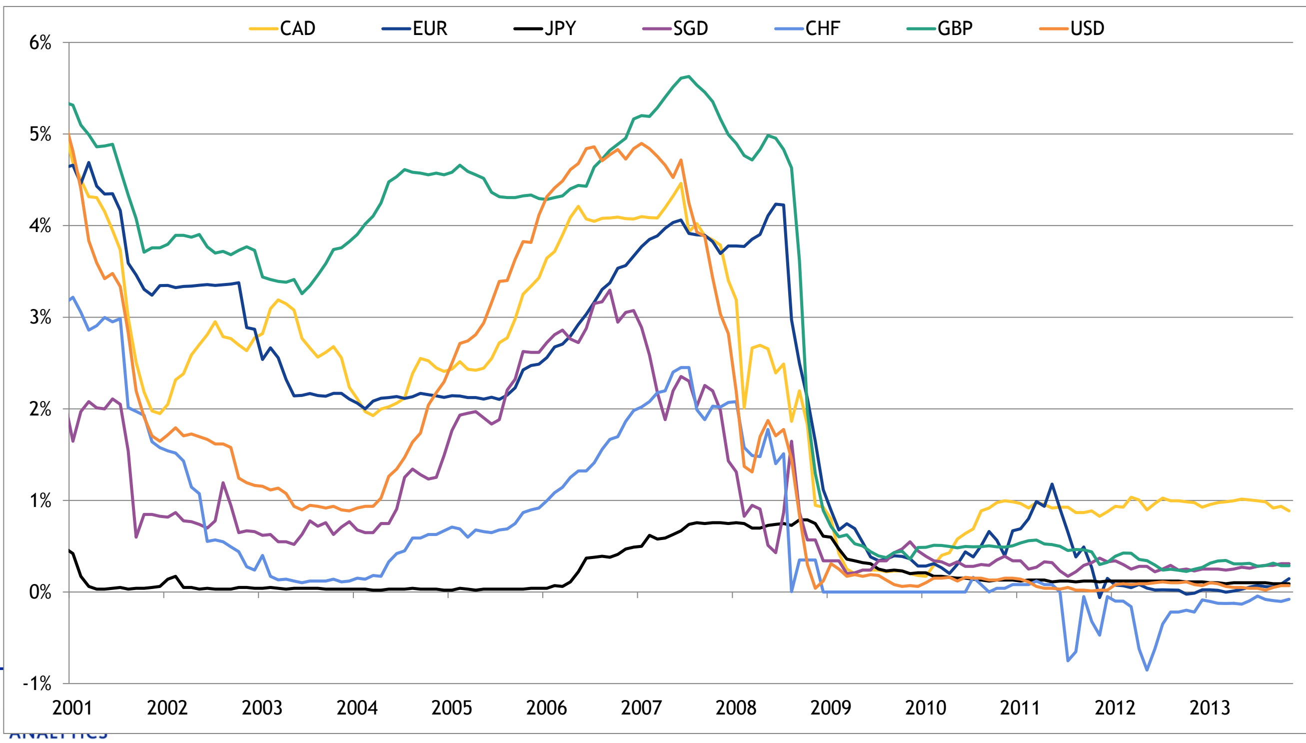
2007/2008 financial crisis and subsequent recession (2)

» Deflationary shocks to global economic output



2008 financial crisis and subsequent recession (3)

- Unprecedented policy response
  - Extreme rate cuts and “quantitative easing”
- Significant change in central bank rhetoric and policy



Quantitative easing

The Bank of England:

“The purpose of asset purchases is to inject money directly into the economy in order to boost nominal demand.”

“The Bank of England electronically creates new money and uses it to purchase gilts from private investors…they tend to use it to purchase corporate bonds and shares…that lowers long term borrowing and encourages the issuance of new equities and bonds to stimulate spending and keep inflation on track to meet the government’s target”

The Federal Reserve:

“…the Fed’s asset purchases are a monetary policy tool that the Fed has used to stimulate the economy since 2008…the Fed will begin to tighten monetary policy at the appropriate time to avoid any emerging threat to price stability.”

Inflation is still low

Riksbank:

"The broad downturn in inflation and the repeated downward revisions to the inflation forecast imply that underlying inflationary pressures are very low and lower than previously assessed,"

Research and development

Focus on model performance in capturing

- Persistent low interest rate environment
- Probability of negative or floored interest rates

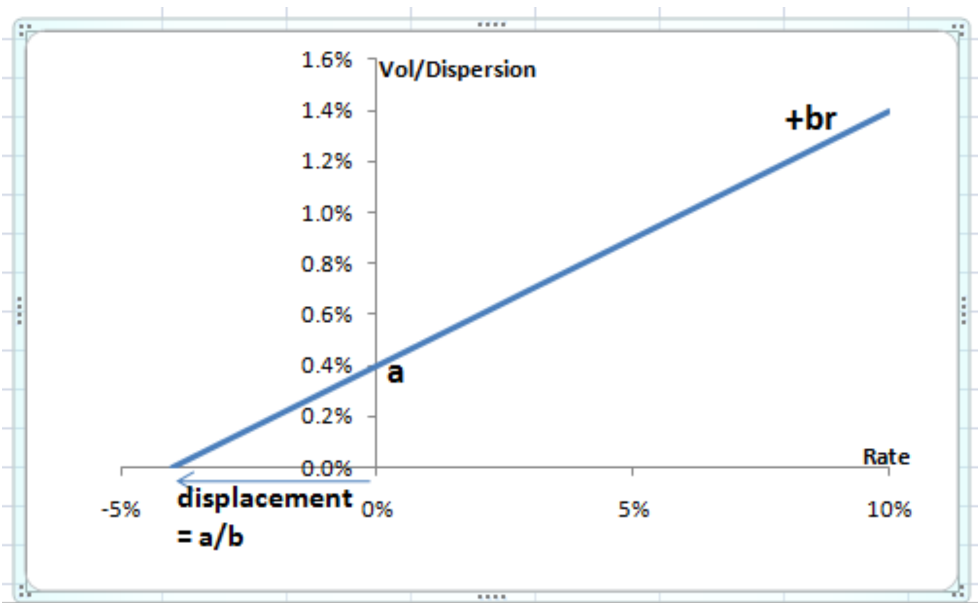
We have developed and calibrated *displaced lognormal* interest rate models

- The LMM + model for 1Y VaR applications
  - More recent research into extending the premia functionality for Multi-Year modelling

- The Extended Displaced 2 Factor Black-Karasinski model for Multi-Year applications
  - Through-the-cycle model and calibration

Displaced Models

The model allows user more flexibility to model the dependency between rate volatility and rate levels:



Displacement (1)

Estimation and calibration

Estimated/calibrated by examining the relationship between interest rates and conditional volatility

- Conditional volatility measured by 1 month options on 1 to 10 year swaps
- Relationship between swap rates and conditional volatility investigated

Volatility $\sigma_{i,k}$, for each k maturity rate in each economy i, in a displaced, δ, lognormal model:

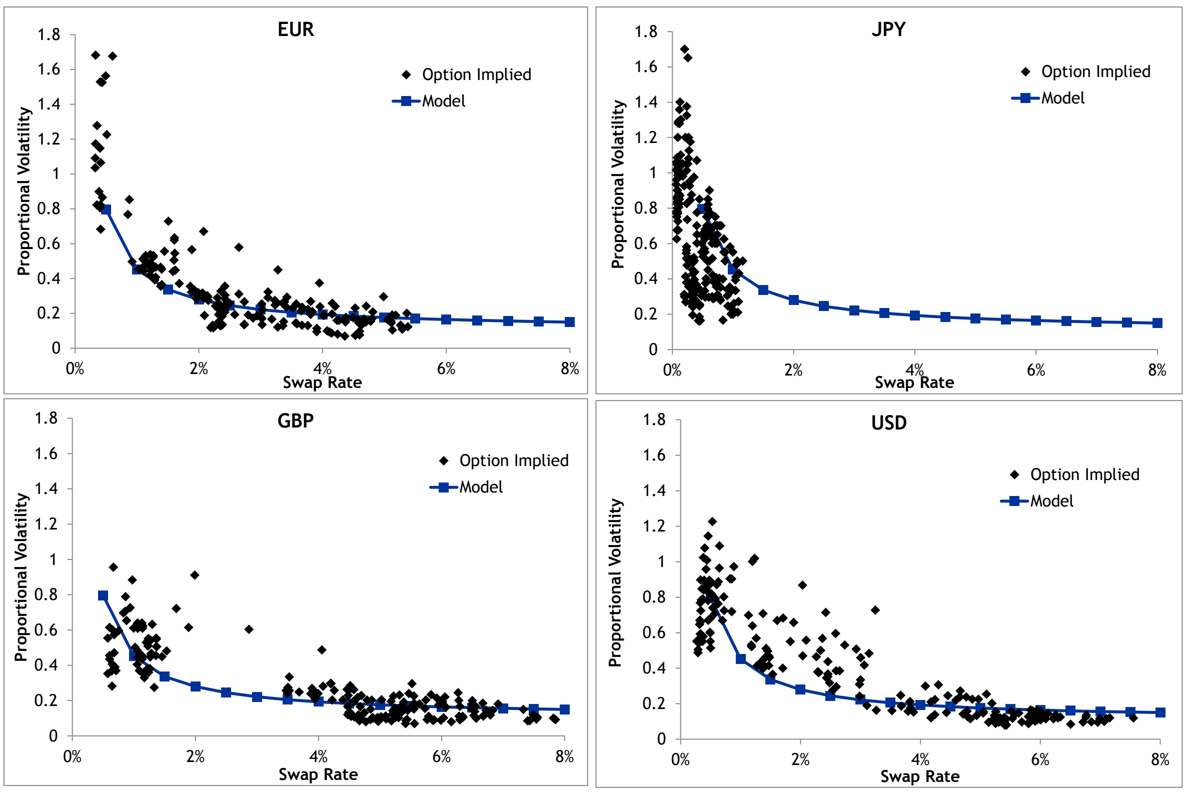
$$\frac{\sigma_{\boldsymbol{i},k}\left(f_{\boldsymbol{i},k}(t),t\right)}{f_{\boldsymbol{i},k}(t)} = g_{\boldsymbol{i},k} + g_{\boldsymbol{i},k}\frac{\delta}{f_{\boldsymbol{i},k}(t)}$$

$$\frac{\sigma_{\boldsymbol{i},k}\left(f_{\boldsymbol{i},k}(t),t\right)}{f_{\boldsymbol{i},k}(t)} f_{\boldsymbol{i},k}(t) = \mathrm{IV}_{i,k}.\, f_{\boldsymbol{i},k}(t) = g_{\boldsymbol{i},k}\left(f_{\boldsymbol{i},k}(t)+\delta\right)$$

- Can be estimated by regression (swaption implied volatility, $\mathrm{IV}_{i,k}$, can be observed
- Estimate δ to minimize the ‘coefficient of variation’ between $g_{\boldsymbol{i},k}$ for different economies (soft constraint on global assumption that these are equal.

Displacement (2)

Ĺ	3.22%



Extended displaced two factor Black-Karasinski

The **E(D)2FBK** model

$$r = r' - \delta$$

$$dln\big(r'(t)\big) = \alpha_1\big[ln\big(m(t)\big) - ln\big(r'(t)\big)\big]dt + \sigma_1(dW_1 + \gamma_1(t)dt)$$
$$dln\big(m(t)\big) = \alpha_2\big[\mu'(t) - ln\big(m(t)\big)\big]dt + \sigma_2(dW_2 + \gamma_2(t)dt)$$

- Short rate model; yield curve priced by taking risk neutral expectation of the short rate path:

$$P(t,T) = E^Q\left[\exp\left(-\int_t^T r ds\right)\right]$$

- Displacement is constant, so:

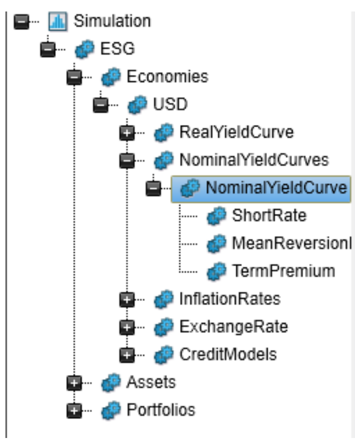
$$P(t,T) = \exp\big(-\delta(T-t)\big)E^Q\left[\exp\left(-\int_t^T r' ds\right)\right]$$

- Implementation into ESG products is relatively straightforward as added functionality in the vanilla E2FBK model

Implementation

Implementation of model is a straightforward extension to existing functionality

- Bond pricing adjusted to take account of displacement term
- Standard E2FBK functionality available (eg direct input yield curve, and ability to target paths of interest rates using time varying term premium functionality)
- If desired, a floor can be set on rates to avoid producing negative rates



Name	Type	Value
ShortRate	OneFactorBK	
MeanReversionLevel	OneFactorBK	
TermPremium	TimeVaryingTermPremiur	
TreeTimestep	Parameter	1y
ShortRateTenor	Parameter	1y
Bond Prices	Parameter Set	(Set: 120 entries)
Modelled Years	Parameter	30
ShortRateCap	Parameter	10
MeanReversionLevelCap	Parameter	10
InitialIndexValue	Parameter	1
Displacement	Parameter	0.0262
BoundingMethod	Parameter	Floor
OutputRateFloor	Parameter	0

Distribution calibration

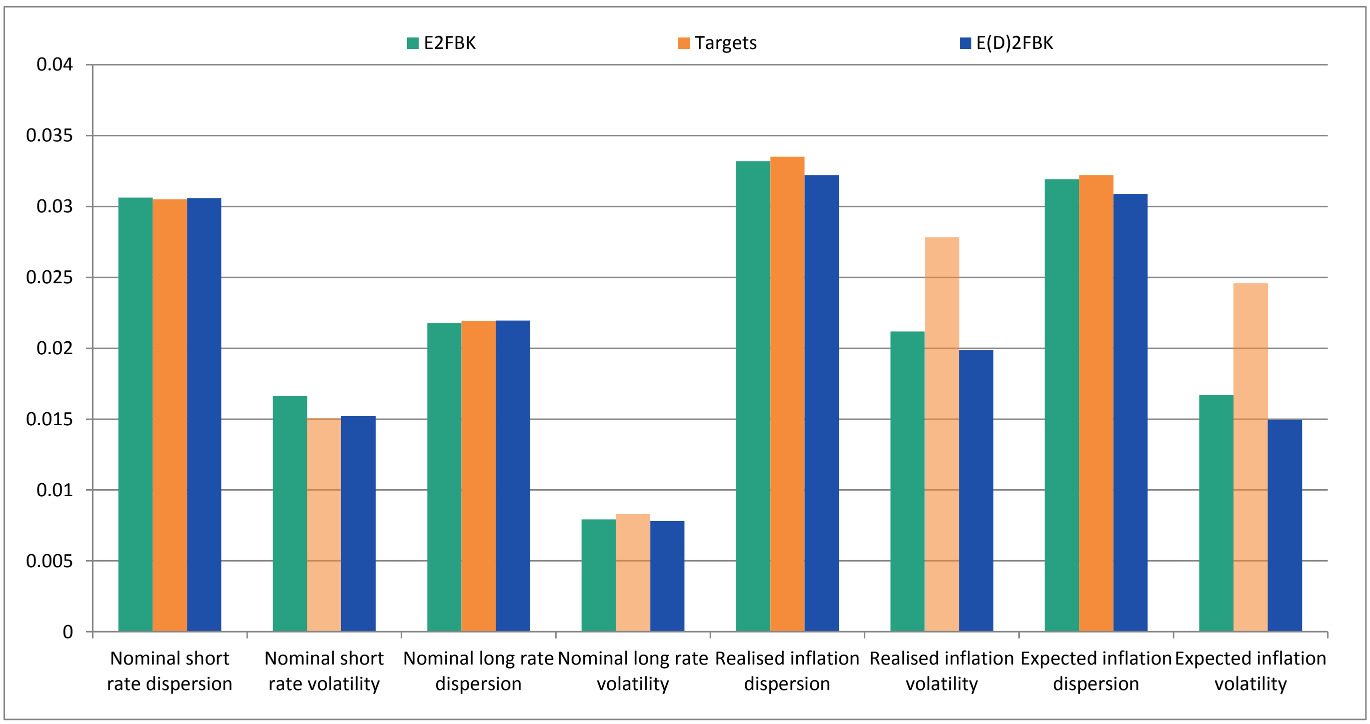
Same approach as standard E2FBK model

- Calibrate model to assumptions for dispersion and correlation
- Validate other features of the distribution as implied by the model

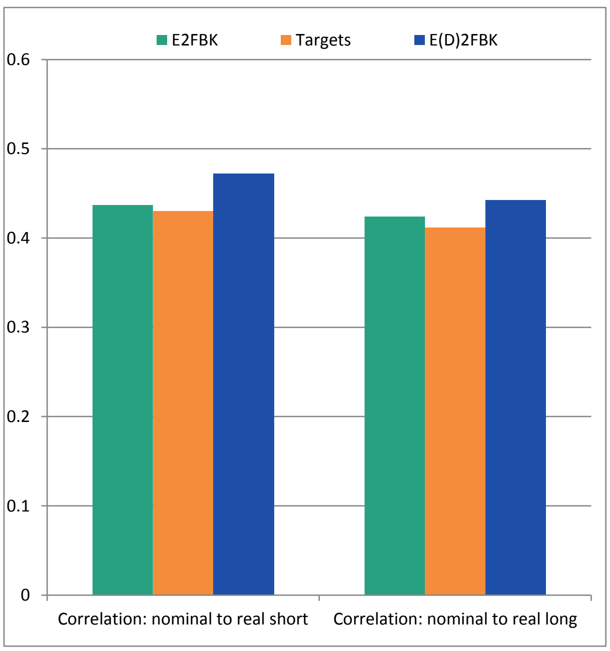
Nominal rate models	E2FBK	E(D)2FBK
Alpha1	0.285	0.245
Alpha2	0.069	0.069
Sigma1	0.346	0.200
Sigma2	0.201	0.117
Displacement	NA	0.032

Validation 2014		Target (%)		E2FBK		E(D)2FBK	
		Dispersion	Volatility	Dispersion	Volatility	Dispersion	Volatility
Nominal	Short Rate	3.05	1.51	3.06	1.66	3.06	1.52
	Long Rate	2.19	0.83	2.18	0.79	2.20	0.78
Inflation	Realised	3.35	2.78	3.32	2.12	3.22	1.99
	Expected	3.22	2.46	3.19	1.67	3.09	1.49

Calibration to targets (1)



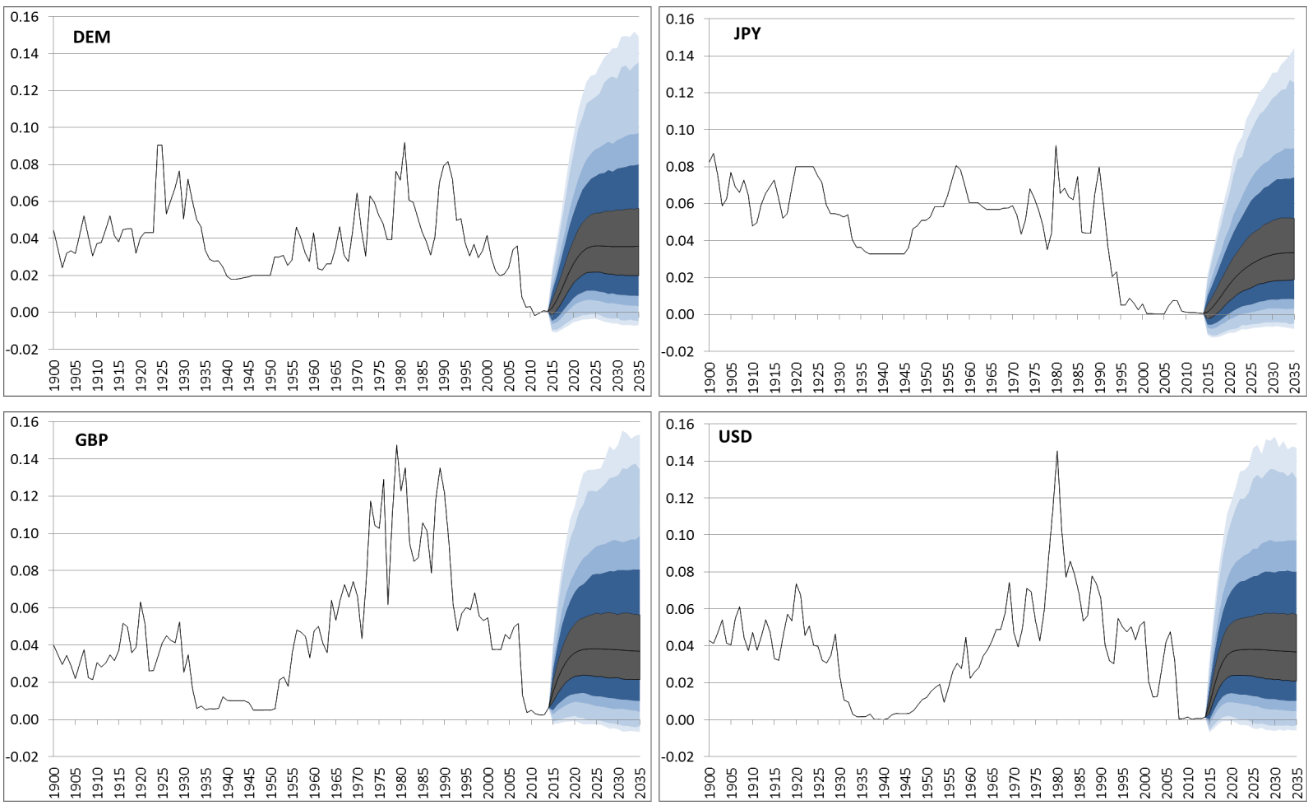
Calibration to targets (2)



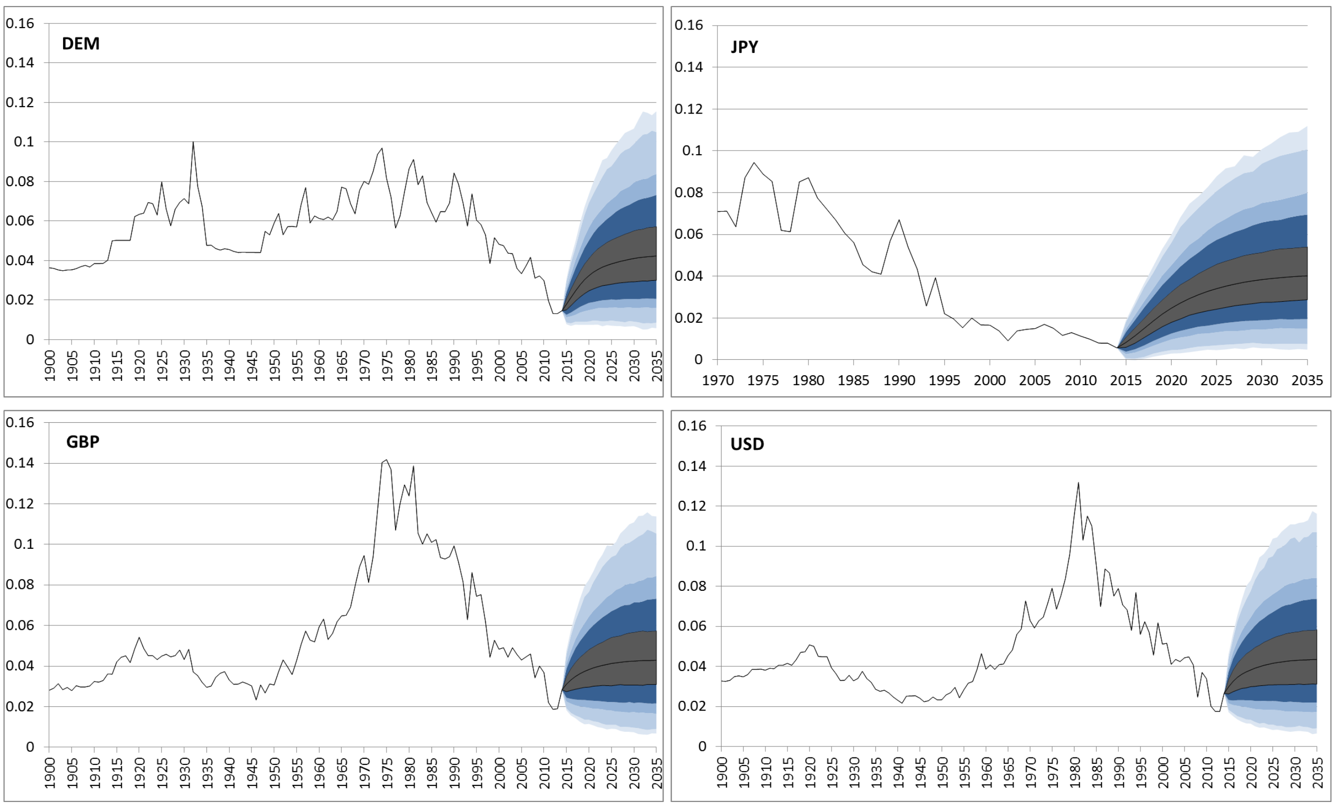
Real to Nominal Correlation	Target	Validation	
		E2FBK	E(D)2FBK
Short	43.00	43.69	47.21
Long	41.16	42.41	44.24

Validation	Levels			Changes		
	E(D)2FBK	E2FBK	Target	E(D)2FBK	E2FBK	Target
1 to 5	96.85	95.62	96.32	95.71	93.81	71.90
1 to 10	90.67	88.32	90.72	83.52	78.76	50.38
1 to 15	86.10	83.61	86.30	72.17	66.75	39.36
5 to 10	98.29	98.10	98.19	95.75	94.87	90.36
5 to 15	95.98	95.78	95.52	88.80	87.48	77.08
10 to 15	99.50	99.53	99.18	98.28	98.27	92.49

Projections and history, 1Y rates



Projections and history, 10Y rates



Shape of the distribution

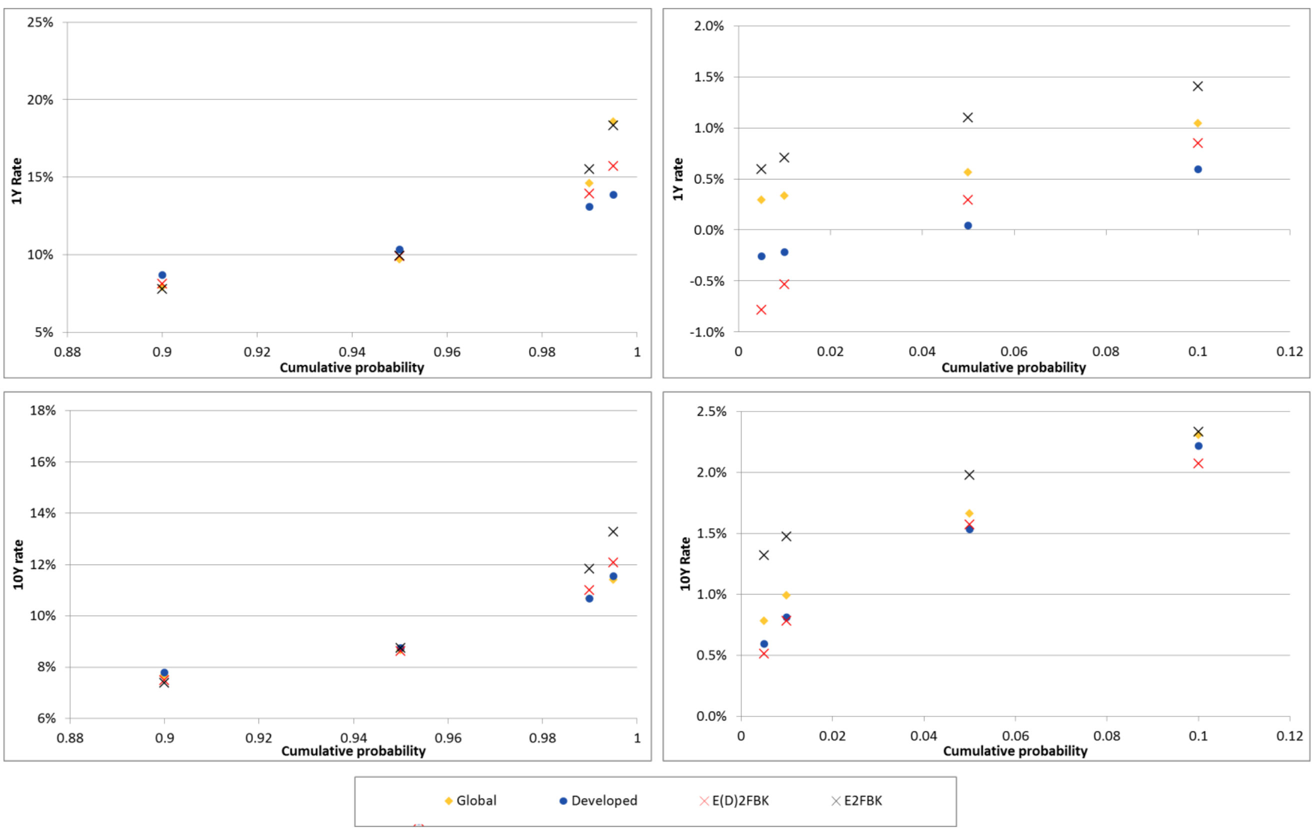
Percentiles and higher moments

- Displacement parameter allows greater flexibility over the shape of the distribution
- No explicit targets or assumptions for the higher moments/percentiles, but useful to compare to different distribution assumptions and empirical evidence

Short Rates		Mean	Standard Deviation	Skew	Kurtosis	Percentiles										
						0.995	0.99	0.95	0.9	0.75	0.5	0.25	0.1	0.05	0.01	0.005
Pooled	Global	5.9%	4.8%	2.1	8.3	4.73	3.43	1.81	1.24	0.42	-0.24	-0.65	-1.01	-1.17	-1.24	-1.26
	Developed	5.2%	3.7%	1.0	0.8	3.19	2.93	2.03	1.48	0.52	-0.20	-0.68	-1.16	-1.34	-1.42	-1.44
	Emerging	8.3%	6.8%	1.8	4.8	3.82	3.29	1.91	1.22	0.44	-0.22	-0.71	-0.97	-1.08	-1.21	-1.21
ESG Validation	E(D)2FBK	4.1%	3.1%	1.3	3.2	3.79	3.21	1.88	1.30	0.50	-0.18	-0.71	-1.07	-1.26	-1.53	-1.61
	E2FBK	4.1%	3.1%	2.6	13.9	4.65	3.72	1.89	1.19	0.34	-0.27	-0.66	-0.89	-0.99	-1.12	-1.16
Std Normal		0	1	0	0	2.58	2.33	1.64	1.28	0.67	0.00	-0.67	-1.28	-1.64	-2.33	-2.58

Long Rates		Mean	Standard Deviation	Skew	Kurtosis	Percentiles										
						0.995	0.99	0.95	0.9	0.75	0.5	0.25	0.1	0.05	0.01	0.005
Pooled	Global	6.6%	3.3%	1.3	5.3	3.12	2.78	1.87	1.42	0.53	-0.20	-0.70	-1.04	-1.34	-1.65	-1.74
	Developed	6.5%	3.1%	0.8	0.6	3.19	2.79	1.90	1.46	0.57	-0.18	-0.71	-1.08	-1.40	-1.73	-1.83
	Emerging	7.3%	4.1%	2.1	10.9	3.56	2.37	1.66	1.25	0.52	-0.17	-0.71	-0.94	-1.18	-1.45	-1.48
ESG Validation	E(D)2FBK	4.6%	2.2%	0.9	1.3	3.43	2.94	1.85	1.33	0.56	-0.14	-0.71	-1.15	-1.38	-1.74	-1.87
	E2FBK	4.6%	2.2%	1.5	4.4	3.98	3.32	1.90	1.28	0.47	-0.21	-0.70	-1.03	-1.20	-1.43	-1.50
Std Normal		0	1	0	0	2.58	2.33	1.64	1.28	0.67	0.00	-0.67	-1.28	-1.64	-2.33	-2.58

Validation



1Y Rate
25%
20%
15%
10%
5%
0.88
0.9
0.92
0.94
0.96
0.98
1
Cumulative probability
1Y rate
2.0%
1.5%
1.0%
0.5%
0.0%
-0.5%
-1.0%
0
0.02
0.04
0.06
0.08
0.1
0.12
Cumulative probability
10Y rate
18%
16%
14%
12%
10%
8%
6%
0.88
0.9
0.92
0.94
0.96
0.98
1
Cumulative probability
10Y Rate
2.5%
2.0%
1.5%
1.0%
0.5%
0.0%
0
0.02
0.04
0.06
0.08
0.1
0.12
Cumulative probability
Global
Developed
E(D)2FBK
E2FBK

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