Bootstrap ex-post reserve as object reserve

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I held a lecture at the Swedish Actuarial Association on 2013-10-22, describing my article Bootstrapping Individual Claim Histories, ASTIN Bulletin 2012(1), 291-324. A question was asked whether the mean bootstrap ex-post reserve

$$R_i^{(-)} = \frac{1}{B}\sum_{t=1}^{B} R_i^{(\nu_t)}$$

given in expression (6.8) on p. 304 could be used as object reserve. It could, but its expectation and almost sure limit as $B \to \infty$ would be better. And that can be computed. It is a PPCF (expected *Payments Per Claim Finalized*) reserve, since all claims in the set Z used for bootstrap must be finalized. Furthermore, they must all belong to claim periods where all claims are finalized. Newly occurred and finalized claims cannot be used. Thus the MSEP will be larger than necessary. In contrast, the chain ladder, Schnieper and RDC methods treated in the paper all use payments on non-finalized claims as well.

To compute this PPCF reserve, consider the historic claim set Z with K claims defined in Section 2.1, p. 294, and pretend that they all occurred in claim period $i \in \{1, \ldots, n\}$. For simplicity, assume no segmentation and no inflation. Also assume that $G^{(\nu)}$ of Assumption A5 is the whole sample space of experiment ν, so that $\nu_t = t$. Define

$$E_i = \sum_{r=1}^{K} \sum_{j>n-i+1} Y(r,j) = \text{total ex-post reserve in } Z,$$

$$K_i = \sum_{r=1}^{K} \mathbf{1}_{\{W(r) \le n-i+1\}} = \text{number of claims reported 'now' in } Z.$$

In the BICH bootstrap procedure we draw a random number $N_i^{(\nu)}$ of claims until M_i are reported 'now'. Here M_i is the number of object claims reported now, see (2.4) p. 295. Then obviously $N_i^{(\nu)}$ has the negative binomial distribution $\mathrm{NB}(M_i, K_i/K)$, with mean $M_i K/K_i$.

For each randomly drawn claim, pretended to have occurred in claim period i, its expected ex-post reserve is of course E_i/K. Since $N_i^{(\nu)}$ is a stopping time (see overleaf for definition), by Wald's theorem we thus obtain

$$\mathrm{E}[R_i^{(-)}] = \mathrm{E}[R_i^{(\nu)}] = \mathrm{E}[N_i^{(\nu)}]E_i/K = \frac{M_i K}{K_i}\frac{E_i}{K} = \frac{M_i E_i}{K_i}.$$

This is the PPCF reserve that could be used. It is unbiased but not suitable. The bootstrap sample Z of claims is just a sample of the real claim distribution.

By $N_i^{(\nu)}$ being a stopping time is meant that the event $\{N_i^{(\nu)} = k\}$ is independent of the sequence of random drawings (if they were to continue) n:os $k+1,\ k+2,\ k+3\ldots$.

A simple account

Pretending that all claims in Z occurred in period i, we can compute the remaining payment sum E_i and the number K_i of claims reported 'now', i.e. known. Then E_i/K_i is the mean reserve per known claim. Hence

$$\text{(number of known object claims)} \times \text{(mean reserve per known claim)} = M_i E_i / K_i$$

is the object reserve with this PPCF method.

Available claims

There are not more claims available for these reserves, since we could include the claims used for bootstrap also in the set Tused for object (actual) reserves. I state on p. 294, line 4 from bottom, that "The claims for i that are finalized could be part of Z." The converse is also true, namely that the claims of Z could be part of T. We do not always, however, want to include all claims of Z in T, due to practice changes.

Addendum to paper

In (6.10) I gave the standard deviation estimate

$$\widehat{D}[R_i^{(\nu_1)}] = \sqrt{\frac{1}{B-1}\sum_{t=1}^{B}\left(R_i^{(\nu_t)} - R_i^{(-)}\right)^2}.$$

Since the mean is known, a better estimate is

$$D^*[R_i^{(\nu_1)}] = \sqrt{\frac{1}{B}\sum_{t=1}^{B}\left(R_i^{(\nu_t)} - \frac{M_i E_i}{K_i}\right)^2},$$

if $G^{(\nu)}$ of Assumption A5 is the whole sample space of experiment ν.