

# The impact of the underlying interest rate model

- when calculating the best estimate of liabilities

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# Agenda

- Introduction
- Interest rate models
- Analysis of scenario files
- Impact on the TVOG
- Summary and Conclusion

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# Introduction

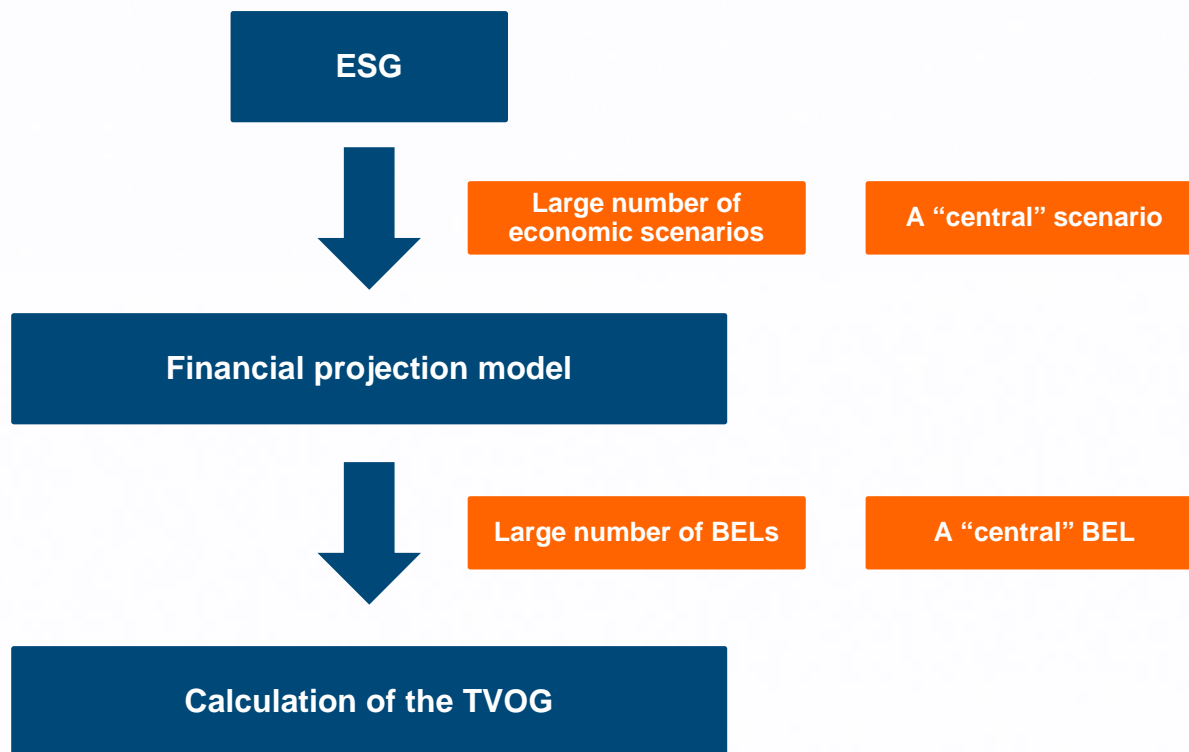
- Time value of options and guarantees.
  - Time value of the options and guarantees (TVOG) is required under Solvency II.
  - The purpose of the TVOG is to reflect the value of the uncertainty of the obligations the insurance company has taken by promising future guaranteed amounts to their policyholders.
  - These guarantees creates an option value due to the asymmetry in the contracts.

# Introduction

- TVOG – An example
  - An insurance contract with profit participation.
  - If the insurance company earns an investment return which is in excess of the guaranteed amount, the policyholder will get a discretionary amount in addition to the guaranteed. However, if the investment return is insufficient to cover the guaranteed amount; the insurance company has to cover the loss.

# Introduction

- How to determine the TVOG?



# Introduction

- How to determine the TVOG?

ESG



- Which underlying model is used?
  - Cox–Ingersoll–Ross
  - Hull and White
  - Libor Market Model

- ▶ **Does it matter? Will they all give the same value?**

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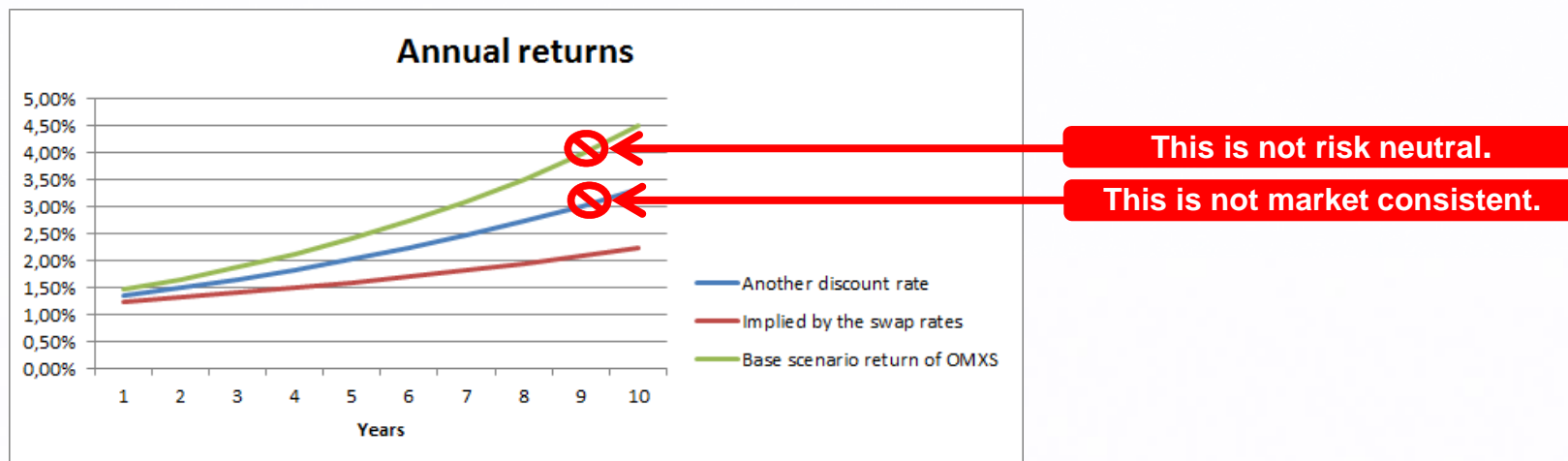
# Interest rate models

Question	Cox–Ingersoll–Ross	Hull and White	Libor Market Model
Used in Nordic market?	YES	YES	YES
Includes negative rates?	NO	YES	NO
Has a fix volatility term?	YES	NO	NO

- There are several different modifications and calibrations conditions of the models, the answers above refers to models used in this work.

# Interest rate models

- The two main requirements for scenarios used to determine the TVOG is that they are market consistent and risk neutral.



- All of these models fulfill these demands to a certain extent.

# Agenda

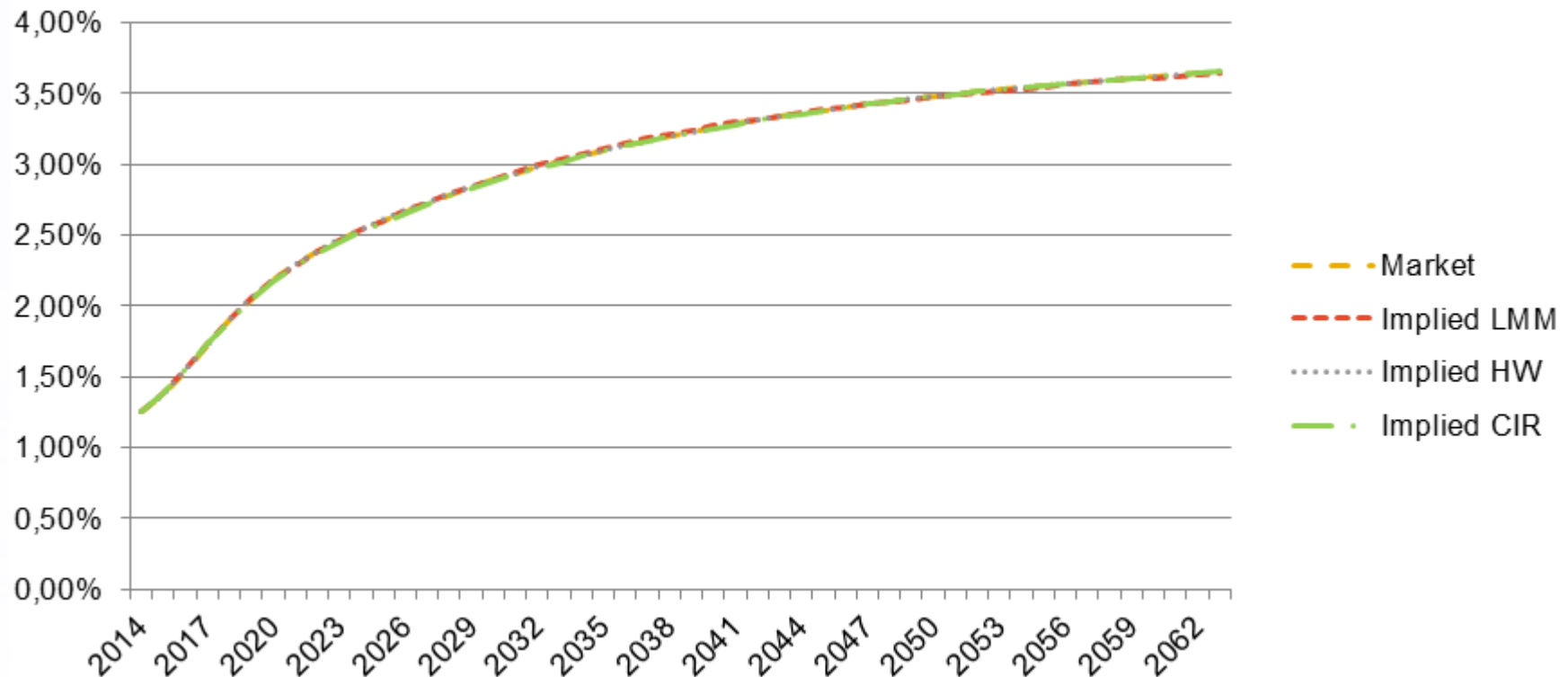
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# Analysis of scenario files

- The scenario sets used in this work consists of 3 000 scenarios.
- The scenario sets must satisfy a number of conditions in order to be risk neutral and market consistent, e.g. :
  - Are all investment strategies “in average” equivalent to a risk-free investment?
  - Is the yield curve derived from the average of discount factor equal to the market yield curve?
  - Do the scenarios replicate swaption prices?
  - Do the scenarios replicate index asset-option prices?
  - Is the correlation reflected correctly?
  - Do the scenarios reflect the markets expectations (e.g. inflation)?

# Analysis of scenario files

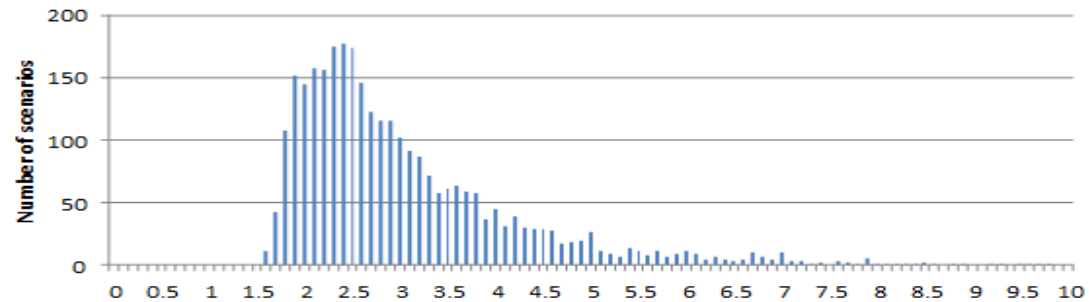
- Implied spot rates by averaging the discount factors



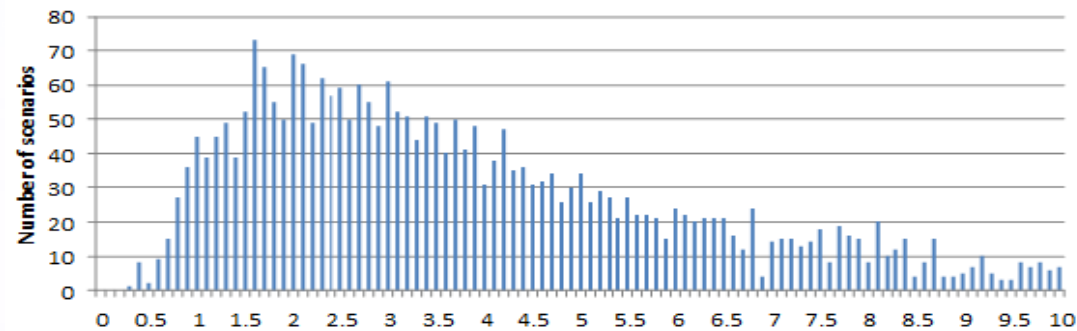
# Analysis of scenario files

- Distribution of a short rate index after 30 years

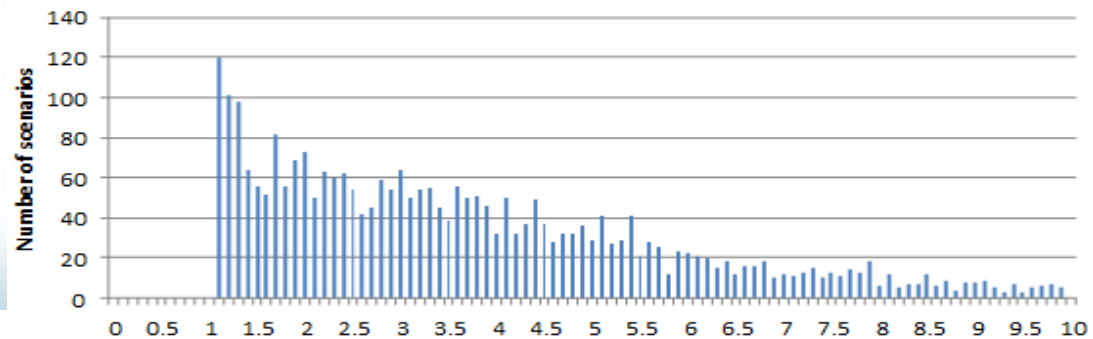
CIR



HW



LMM



# Analysis of scenario files

- Are the market prices replicable?

CIR

Total error (%) - Market price vs Scenario price														
Option Expiry (Years)	Swap Tenor (Years)													
	1	2	3	4	5	6	7	8	9	10	15	20	25	30
1	-10.62%	-30.79%	-39.47%	-41.39%	-41.80%	-40.23%	-37.94%	-35.55%	-31.98%	-28.41%	-7.20%	7.53%	17.40%	24.50%
2	-34.94%	-42.13%	-43.33%	-42.40%	-40.65%	-37.36%	-33.57%	-29.14%	-23.95%	-18.05%	10.27%	28.07%	37.97%	43.46%
3	-46.72%	-46.04%	-44.44%	-41.46%	-37.61%	-32.91%	-27.56%	-21.14%	-15.08%	-8.57%	23.00%	40.48%	47.93%	50.64%
4	-49.89%	-48.45%	-45.17%	-40.14%	-34.08%	-27.58%	-20.03%	-12.72%	-5.70%	1.11%	33.50%	48.42%	52.15%	53.17%
5	-50.05%	-47.82%	-43.19%	-37.03%	-29.23%	-21.07%	-12.43%	-4.41%	3.30%	10.49%	42.74%	55.48%	56.99%	55.32%
7	-48.01%	-46.03%	-40.13%	-30.91%	-19.37%	-9.79%	-0.75%	7.52%	14.81%	21.50%	48.67%	60.50%	58.48%	53.43%
10	-47.41%	-42.70%	-34.62%	-24.12%	-11.41%	1.16%	12.81%	23.06%	31.96%	39.56%	68.40%	70.91%	67.47%	60.85%
15	-51.61%	-45.62%	-35.08%	-22.28%	-8.51%	4.03%	16.12%	26.49%	35.06%	42.33%	65.96%	67.27%	63.05%	56.80%
20	-56.79%	-52.06%	-38.22%	-21.85%	-6.45%	6.65%	17.15%	28.16%	36.93%	44.55%	64.58%	71.23%	65.05%	58.02%
25	-61.80%	-53.10%	-36.26%	-19.16%	-2.92%	10.96%	20.36%	31.64%	40.70%	48.68%	74.30%	76.76%	68.29%	60.33%
30	-62.07%	-49.48%	-27.57%	-6.86%	12.05%	28.71%	39.12%	51.60%	61.37%	69.14%	90.90%	87.25%	76.00%	61.85%

HW

Total error (%) - Market price vs Scenario price														
Option Expiry (Years)	Swap Tenor (Years)													
	1	2	3	4	5	6	7	8	9	10	15	20	25	30
1	39.64%	8.15%	-6.03%	-10.32%	-12.68%	-12.55%	-11.85%	-11.45%	-9.85%	-8.60%	-1.70%	-2.16%	-4.20%	-5.74%
2	17.00%	3.07%	-1.77%	-4.13%	-6.08%	-6.34%	-6.51%	-6.25%	-5.37%	-3.93%	1.16%	0.08%	-2.05%	-3.56%
3	-0.40%	-0.50%	-1.15%	-1.60%	-2.00%	-2.30%	-2.58%	-1.99%	-2.03%	-1.61%	1.81%	0.53%	-1.84%	-3.43%
4	-2.99%	-2.43%	-1.74%	-0.73%	-0.02%	-0.09%	0.48%	0.31%	-0.19%	-0.64%	1.46%	-0.75%	-3.97%	-5.08%
5	-1.70%	-0.83%	0.42%	1.21%	2.28%	2.39%	2.43%	1.69%	0.98%	0.32%	1.88%	-0.42%	-3.31%	-4.49%
7	-2.89%	-4.10%	-3.10%	-0.82%	1.42%	-0.02%	-1.83%	-3.55%	-5.19%	-6.39%	-7.03%	-6.86%	-8.75%	-9.52%
10	0.39%	2.51%	2.69%	1.67%	1.18%	0.34%	-0.43%	-1.13%	-1.77%	-2.36%	-0.40%	-2.16%	-1.42%	-0.43%
15	6.91%	6.83%	4.95%	1.89%	-0.19%	-1.43%	-1.39%	-1.41%	-1.59%	-1.60%	2.14%	4.48%	8.35%	11.58%
20	-1.29%	-7.04%	-6.98%	-7.66%	-7.59%	-7.88%	-8.63%	-7.53%	-6.87%	-5.89%	-0.32%	8.99%	14.01%	17.69%
25	-10.23%	-10.90%	-11.02%	-11.52%	-10.71%	-10.24%	-11.52%	-9.84%	-8.47%	-6.71%	5.56%	15.41%	20.46%	23.91%
30	-11.16%	-9.82%	-8.43%	-7.40%	-5.97%	-4.48%	-5.59%	-3.50%	-1.73%	0.08%	12.95%	22.44%	28.04%	28.16%

LMM

Total error (%) - Market price vs Scenario price														
Option Expiry (Years)	Swap Tenor (Years)													
	1	2	3	4	5	6	7	8	9	10	15	20	25	30
1	8.72%	0.27%	-4.11%	-4.62%	-5.38%	-4.16%	-2.87%	-2.42%	-1.12%	-0.39%	3.97%	1.54%	-2.16%	-5.54%
2	4.18%	0.57%	-0.71%	-1.96%	-3.57%	-3.83%	-4.44%	-5.05%	-5.08%	-4.49%	-2.56%	-5.41%	-9.05%	-12.23%
3	-3.87%	-0.56%	-0.04%	-0.07%	-0.74%	-1.67%	-3.06%	-3.63%	-4.74%	-5.31%	-5.24%	-8.24%	-11.99%	-15.16%
4	-4.35%	-1.86%	-0.10%	1.02%	1.02%	-0.37%	-1.16%	-2.50%	-4.09%	-5.55%	-7.04%	-10.73%	-14.84%	-17.49%
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7	6.22%	4.51%	4.14%	4.84%	5.56%	2.56%	-0.51%	-3.25%	-5.68%	-7.48%	-9.91%	-11.29%	-14.65%	-17.73%
10	5.30%	7.24%	6.64%	4.73%	3.37%	1.71%	0.37%	-0.76%	-1.68%	-2.49%	-1.86%	-5.47%	-7.73%	-9.72%
15	7.89%	7.49%	4.88%	1.34%	-1.08%	-2.63%	-3.02%	-3.46%	-4.14%	-4.71%	-3.94%	-5.90%	-6.33%	-6.36%
20	5.38%	-1.59%	-2.39%	-4.00%	-4.70%	-5.75%	-7.30%	-6.92%	-6.92%	-6.69%	-6.80%	-3.49%	-2.38%	-1.73%
25	-1.51%	-3.16%	-4.01%	-5.41%	-5.52%	-6.30%	-9.11%	-8.96%	-9.06%	-8.71%	-2.99%	2.28%	3.92%	4.53%
30	-0.15%	-1.39%	-2.13%	-3.04%	-3.52%	-3.89%	-6.62%	-5.89%	-5.42%	-4.69%	3.08%	8.11%	9.59%	7.06%

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# Impact on the TVOG

- A simple guarantee product
  - The policyholder will receive a lump sum payment upon retirement.
  - The lump sum payment will be the maximum of the fund value and the guaranteed value at the time retirement.
  - Three different net guaranteed rate: 0%, 2% and 4%.
  - The entire fund value is assumed to be invested in short duration bonds and credited interest based on the short rate.
  - If the policyholder dies before retirement the lump sum payment occurs at the time of death. In this case the lump sum payment will also be the maximum of the fund value and the guaranteed value.
  - If the policyholder surrenders before retirement, he will only receive the fund value.

# Impact on the TVOG

- With a guaranteed rate of 0%:
  - Only the HW-model generates a TVOG, due to the fact that it is the only model that allows for negative interest rates.
- With a guaranteed rate of 2%:
  - HW generates the highest TVOG for the sample product used.
  - The TVOG with LMM is about 2/3 of the HW value.
  - The CIR-model does not generate any TVOG.
- With a guaranteed rate of 4%:
  - HW still generates the highest TVOG.
  - LMM generates a TVOG just slightly lower than HW.
  - The CIR-model generates a TVOG significantly lower than both HW and LMM.

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# Summary and Conclusion

- The main findings:
  - The choice of interest rate model used for the valuation impacts the TVOG.
  - The differences between the interest rate models seem to be the most significant for products with a guaranteed rate of 0% due to the allowance of negative rates in HW.
  - The interest rate models fail to fully replicate the market prices, especially the CIR.
- It impacts the TVOG:
  - This means that it impacts the best estimate of liabilities.
  - It could potentially impact the management's view of the business.
  - It makes comparisons intransparent between companies (when different models are applied).

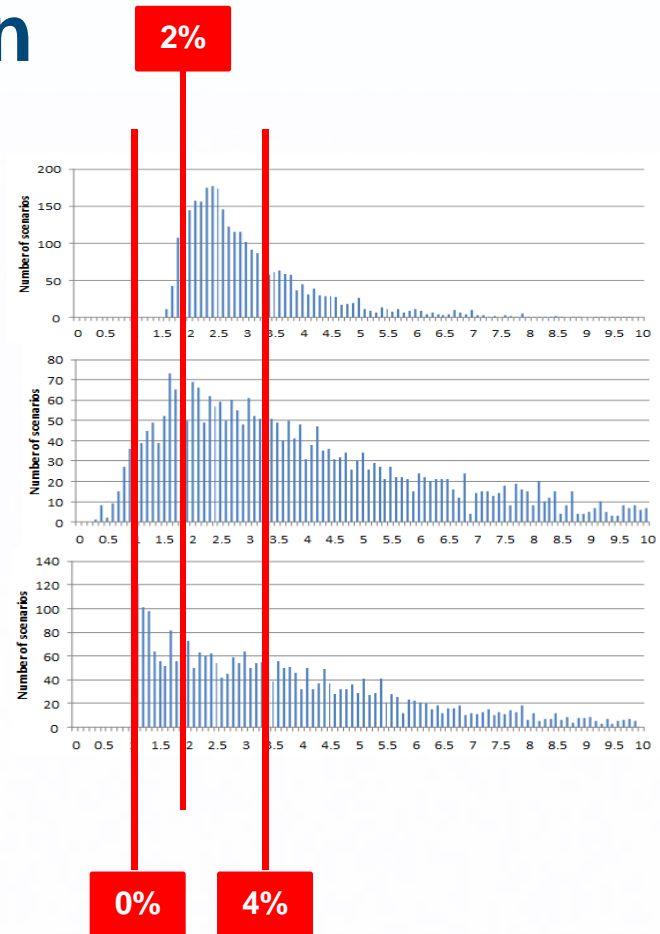
# Summary and Conclusion

- The short rate index after 30y.
- It illustrates and explains the results very well.

CIR

HW

LMM



# Summary and Conclusion

- The main limitations:
  - Everything is invested in short durations bonds.
  - The failure in replicating market prices could potentially create parts of the differences in the TVOG (e.g. HW could have had a different TVOG with a perfect calibration).
  - Simple product setup, different features, such as annuity payments and/or ratcheting could potentially have caused other results.
- Finally, the focus in this work has been to present and illustrate that the choice of interest model is important and will affect the results.

# Questions?

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